

# Robust Scheduling of Electric Vehicle Charging in LV Distribution Networks under Uncertainty

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**Abstract**—Rapid increase in the number of electric vehicles will likely deteriorate voltage profiles and overload distribution networks. Controlling the charging schedule of electric vehicles in a coordinated manner provides a potential solution to mitigate the issues and could defer reinforcement of network infrastructure. This work presents a method for robust, cost-minimising, day-ahead scheduling of overnight charging of electric vehicles in low voltage networks in a stochastic environment with minimal real-time adaptation. To reduce the computational complexity, a linear power flow approximation is utilised. The stochastic environment captures multiple uncertainties arising from the mobility behaviour including stochastic daily trip distances, arrival and departure times. Knowledge about the probability distributions of these parameters is used to hedge risks regarding the cost of charging, network overloading, voltage violation and charging reliability. The results on a test network provide an insight into the impact of uncertainty and the effectiveness of addressing aspects of risk during optimisation. In particular, planning with more conservative estimates of initial battery charge levels increases the reliability and technical feasibility of optimised schedules.

**Index Terms**—Electric vehicles, charging, uncertainty, distribution networks, optimisation.

## I. INTRODUCTION

THE electrification of personal mobility is critical to reduce carbon emissions as well as air pollution in urban areas and is actively supported by governments worldwide. The additional load from increasing levels of uncontrolled residential electric vehicle (EV) charging will have negative impacts across the power system. At whole system level it will necessitate more power plants to supply the power and additional reserve to handle variability. At distribution level, effects include excessive voltage drops, phase imbalances and overloading of network components, especially when many EVs charge simultaneously. Unfortunately, EV charging is likely to cluster when commuters arrive home at the end of the workday and plug-in their EVs. Thus, immediate charging, especially in the evening at times of highest residential load, increases the detrimental impact.

While investment in major network reinforcement and new power plants is an intuitive measure to incorporate additional EV loads, it may be more economically efficient to encourage EV users to charge at off-peak times. Existing distribution networks can accommodate substantial penetration levels of EVs if charging is coordinated. Exploiting the demand-side

flexibility of EVs through scheduling of charging will mitigate negative effects and can promote renewable integration [1].

For users of EVs to engage in and accept coordinated charging, attractive monetary benefits and recharging reliability are essential. Time of use tariffs can provide an incentive to dynamically adjust loads. Well-designed dynamic tariffs based on temporally and geographically resolved day-ahead or intra-day spot markets mirror the general network state. For the small volume of individual EVs to gain access to electricity markets, an intermediary may be necessary to whom individual users delegate remote charging control. This so-called *aggregator* minimises charging costs by harnessing the load flexibility of the EVs while observing downstream network, equipment and demand constraints.

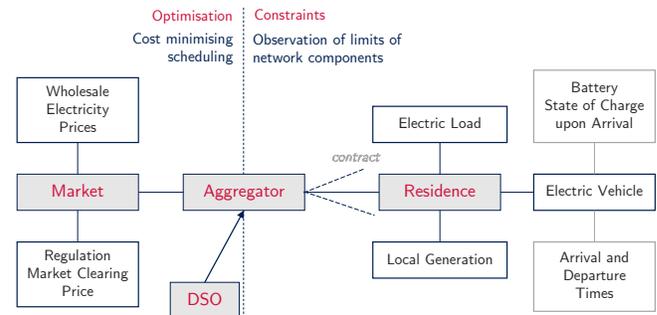


Fig. 1: Overview of sources of uncertainty in EV scheduling

While day-ahead schedules facilitate finding cost-efficient solutions, forecasts of individual consumers' behaviour are prone to substantial deviations. As outlined in Fig. 1, beyond uncertainty of arrival and departure times there is uncertainty around battery state-of-charge (SOC) on arrival, non-EV residential loads, local renewable generation, and wholesale market prices. In light of this, charging coordination must incorporate possible forecast errors into the optimisation [2]. In recent years, a number of papers have presented diverse approaches to tackle uncertainties, primarily addressing spot market prices, variable renewable generation, and individual EV-user behaviour [3], [4].

Further, the literature offers a broad range of methods using stochastic, robust optimisation and receding horizon control approaches. Robust optimisation is predominantly applied to hedge against uncertain spot market prices: [5] develop a scheduling approach based on a bi-level robust optimisation to minimise worst-case price prediction errors. Using a mixed-integer quadratic programming model, [4] show the benefit of aggregators participating in regulation services by reducing the uncertainty of network operation costs. Stochastic optimi-

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sation problems have been solved by information gap decision theory [6], [7], Lyapunov optimisation [8], Markov decision processes [9], [10], Monte Carlo methods [11] and dynamic stochastic optimisation [12]. Receding horizon optimisation is also employed to use prior knowledge and updated forecasts to adapt optimised schedules; this includes sequential quadratic programming [13], alternating direction method of multipliers [14] and model predictive control [15].

Three frequently neglected issues motivate this work. First, among the papers addressing various domains of uncertainty, few focus on the technical constraints of low voltage (LV) distribution networks which are sensitive to EV loads [5], [13]. Second, stochastic and robust optimisation problems add substantial complexity to solution methods; a question arises as to whether simpler, deterministic, optimisation with conservative parameter estimates would facilitate the adoption of coordinated charging schedules while achieving equal levels of robustness. Third, the common focus on isolated aspects of uncertainty and narrow model scope limit comprehensive and realistic understanding of the charging problem under multiple sources of uncertainty; the costs and benefits of robustness have therefore been only partially evaluated.

The main contribution of this work is to develop a robust cost-minimising day-ahead scheduling approach for charging EVs while simultaneously maintaining EV owner satisfaction and local network constraints in a stochastic environment. It is envisaged that schedules are optimised by an aggregator who participates in a wholesale electricity market, and schedules day-ahead for coordination purposes. Another contribution is to derive suitable models for inherent uncertainties associated with the optimal scheduling of EVs. Knowledge about the probability distributions of the availability of EVs for charging and their battery state of charge is used to hedge risks regarding the cost of charging, network overloading, voltage violation and charging reliability. Stochastic input parameters for the scheduling problem can employ more conservative estimates rather than their expectation values. This increases the chances of feasibility of the charging schedule despite parameter deviations and makes the results robust up to a chosen level. Therefore, the contribution of this work is twofold: *stochastic models* of EV uncertainties complement the development of an *optimal scheduling* approach with mitigation options for different levels of robustness.

The remainder of this paper is structured as follows. Sections II to V, respectively, describe the approach for stochastic analysis, the EV charging algorithm, the uncertainty mitigation techniques and the integrated simulation framework. This is followed by a case study and conclusions.

## II. STOCHASTIC MODELS FOR EV DEMAND AND AVAILABILITY

Appropriate behavioural models of daily travel patterns are essential for modelling EV loads and the design of efficient charging coordination. Here, continuous distributions are fitted to empirical data on users' average characteristics and typical deviations from this. The overall process is depicted in Fig. 2 and described below.

### A. Data Source and Extraction

Data sets on the mobility behaviour of EV owners are sparse. Instead, empirical driving profiles of internal combustion vehicles are employed and no systematic change in behaviour by switching transport fuels is assumed. To model the behaviour of customers, this study uses mobility data obtained from the UK National Travel Survey [16]. This extensive annual survey contains disaggregated data on means, demographics and behaviour of personal travel based on diaries dating back to 2002. From this the average  $\mu$  and standard deviation  $\sigma$  of daily mileage, arrival and departure times of individual households, was determined.

The arrival and departure times, respectively, refer to arrival at home after the last trip of the day and departure from home just before the first journey of the next day. For each household, the average and standard deviation of these arrival and departure times among the logged days are determined and compiled as two sets of  $(\mu, \sigma)$ -tuples characterising the required information about a household's EV availability. Daily trip mileage in the data set is the sum of all individual trips for each household during the logged day. This gives the range of modelled EVs as specified by battery capacity. Daily mileage values are recorded as a set of  $(\mu, \sigma)$ -tuples describing a car's typical driving distance.

### B. Statistical Analysis of Empirical Data

Histograms of the acquired  $(\mu, \sigma)$ -tuples are depicted in Fig. 3, emphasising the heterogeneity of both average and variance of parameters among households. In an average household, the first trip by car occurs at 10:30 am, the last trip ends at 16:45 pm with travel totalling 22 miles (35 km). Average deviation in departure/arrival times are 2 and 2.5 hours and 15 miles (24 km) in driving distance.

Fig. 3 includes continuous distributions fitted to the empirical data. The distribution for each parameter is chosen based on the Akaike information criterion resulting in four types of distributions describing the six random variables of the mobility behaviour model, the parameters of which are shown in Fig. 3. Average departure time and daily mileage are described by Gamma distributions which have been applied in other work, e.g. [17]. The standard deviations of departure times and mileage, respectively, follow half normal distribution and exponential characteristics.

### C. Modelling User Behaviour

Many studies use distributions of daily mileage, arrival and departure times for stochastic scenario generation [18]–[20], following random assignment of these parameters to households. Typically these patterns are regarded as immutable.

1) *Scenario Generation*: The parameters that define the mobility behaviour of EV owners are generated through random sampling from their respective distribution functions and iteratively assigned to households in the test network. Normal distributions  $\mathcal{N}(\mu, \sigma)$  are assumed for the troika of  $(\mu, \sigma)$ -tuples defining the stochastic variables daily mileage  $\delta_{mil}$ , arrival time  $\tilde{\tau}_{arr}$  and departure time  $\tilde{\tau}_{dep}$ , e.g.  $\tilde{\delta}_{mil} \sim \mathcal{N}(\mu_{mil}, \sigma_{mil})$ ,  $\hat{\delta}_{mil} = \mathbb{E}[\tilde{\delta}_{mil}] = \mu_{mil}$ . Simulation under uncertainty will treat expected values  $\hat{\delta}_{mil}$ ,  $\hat{\tau}_{arr}$ , and  $\hat{\tau}_{dep}$

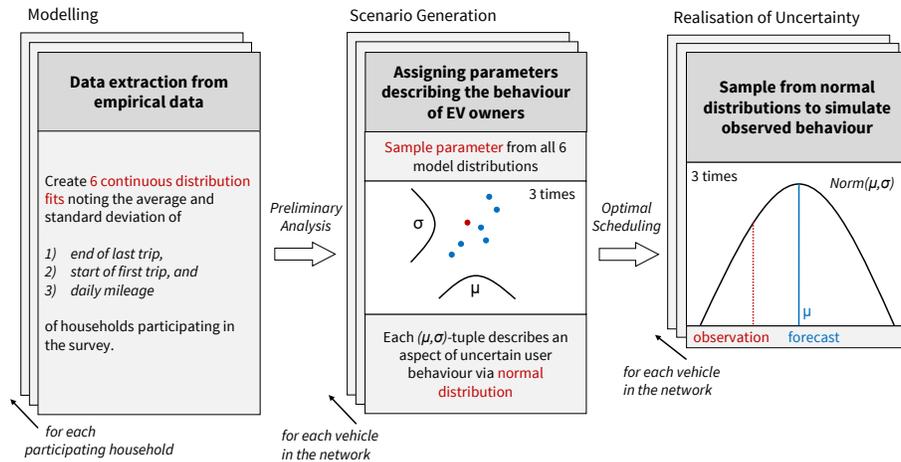


Fig. 2: Illustration of travel pattern model derivation and application

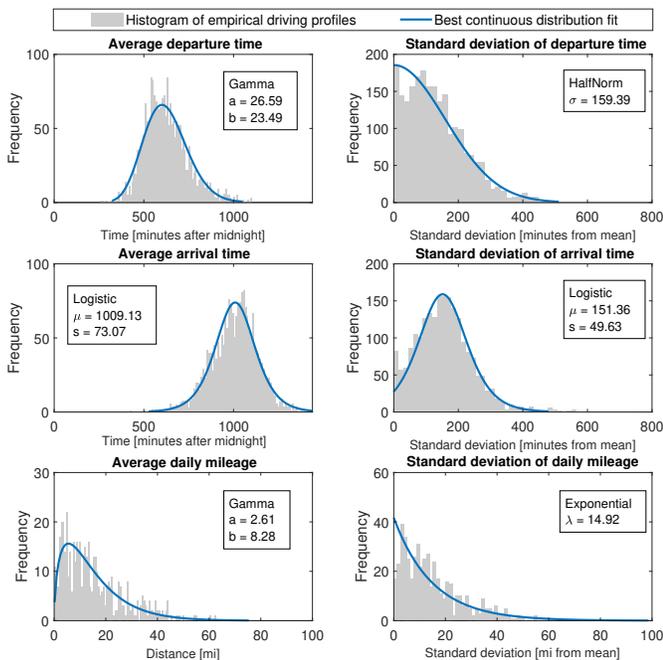


Fig. 3: Marginal distribution fits of  $(\mu, \sigma)$ -tuples of daily mileage, arrival and departure times

as day-ahead forecasts and use randomly chosen values  $\tilde{\delta}_{mil}$ ,  $\tilde{\tau}_{arr}$ , and  $\tilde{\tau}_{dep}$  to emulate the deviation in outturn from the forecast parameters.

2) *EV Charging Demand*: Assuming a full battery at the beginning of the day ( $SOC=B_{max}$ ) and that the user only charges at home overnight, the probability of battery SOC upon arrival  $\tilde{B}^{arr}$  also follows a normal distribution [21] determined by mileage:

$$\tilde{B}^{arr} = B_{max} - \zeta \cdot \tilde{\delta}_{mil} \quad \text{and} \quad \hat{B}^{arr} = B_{max} - \zeta \cdot \hat{\delta}_{mil}. \quad (1)$$

where  $\zeta$  is the EV energy consumption in kWh/km.

3) *EV Availability*: The 'availability'  $\tilde{\alpha}^{EV} \in \mathbb{B}^T$  of an EV describes whether the vehicle is physically plugged into the charger when arriving at home until the car is unplugged prior to departure; during this time control and communication

infrastructure enables the charge rate to be controlled remotely by the aggregator over the optimisation horizon  $T$ . It is a binary parameter and  $\tilde{\alpha}_{k,t}^{EV}$  equals 1 when the EV at household  $k$  at time  $t$  is able to be charged, otherwise it is zero. The availability is calculated from arrival and departure times ( $\tilde{\tau}_{arr}$ ,  $\tilde{\tau}_{dep}$ ), and their conversion to the discrete time slots  $t \in T$  as in Equation 2, where  $\tau_{init}$  is the time of the day in minutes after midnight when the optimisation routine starts and  $\Delta t$  denotes the temporal resolution. Note that if  $\tilde{\tau}_{arr} > \tilde{\tau}_{dep}$  the EV did not return that night and is not available. The expected availability  $\hat{\alpha}^{EV}$  is defined analogously.

The probability  $\mathbb{P}_t(\alpha^{EV} = 1)$  that a vehicle is available in time slot  $t$  is calculated using the cumulative distribution function (CDF) of the standard normal distribution  $\Phi$  as given in Equation 3 (the first and second terms refer to the probability of an EV's arrival or departure). Examples of availability probability curves are shown in Fig. 4 for individual EVs and an aggregate. EV availability is most likely in the early morning and reduces towards noon. The obtained probability curves are also used for the mitigation of availability uncertainty in Section IV-A.

### III. EV CHARGING OPTIMISATION UNDER UNCERTAINTY

The objective is to find the optimal schedule of charging rates for each EV ( $P^{EV} \in \mathbb{R}^{K \times T}$ ) which minimises the total costs of charging the pool of EVs  $k \in K$  in discrete time slots  $\Delta t$  based on a purchasing power at time-dependent price  $\pi_t$ . It is a customer-focused objective providing an incentive for EV users to devolve charging control which enables the aggregator to exploit the available network capacity, and avoiding or postponing network reinforcement.

$$\min_{\{P^{EV}\}} C = \sum_{t=1}^T \sum_{k=1}^K \pi_t \cdot \Delta t \cdot P_{k,t}^{EV} \quad (4)$$

The optimisation is deterministic and employs forecasts of EV behaviour based on statistical estimates from its stochastic models. The presence of network power flow equations makes the problem nonlinear but a linear power flow approximation is applied to simplify the optimisation.

$$\tilde{\alpha}_t^{EV} = \begin{cases} 1 & \text{if } \lfloor \max(0, \frac{\tilde{\tau}_{arr} - \tau_{init}}{\Delta t \cdot 60}) \rfloor \leq t < \lfloor \min(T, T + \frac{\tilde{\tau}_{dep} - \tau_{init}}{\Delta t \cdot 60}) \rfloor \\ 0 & \text{else} \end{cases} \quad \forall t \in T \quad (2)$$

$$\mathbb{P}_t(\alpha^{EV} = 1) = \min \left\{ \Phi \left( \frac{(t \cdot \Delta t \cdot 60 + \tau_{init}) - \mu_{arr}}{\sigma_{arr}} \right), \Phi \left( \frac{(t \cdot \Delta t \cdot 60 + \tau_{init} - T \cdot \Delta t \cdot 60) - \mu_{dep}}{\sigma_{dep}} \right) \right\} \quad \forall t \in T \quad (3)$$

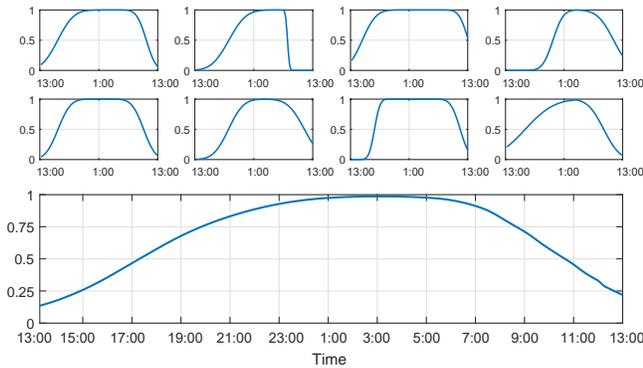


Fig. 4: Availability probabilities for individual EVs (top) and an aggregate of 55 EVs (bottom). Time series begin at 1pm.

The optimisation is subject to a number of constraints, split into those concerning (i) EV technical limitations, (ii) satisfaction of users' requirements and (iii) network-related constraints.

The charging rate is constrained by the limits of the charging mode. As the EV battery is not allowed to discharge in the charging period, the constraint is:

$$0 \leq P_{k,t}^{EV} \leq P_{max}^{EV}, \quad (5)$$

where for standard single-phase connections,  $P_{max}^{EV} = 3.7$  kW.

An EV may only be scheduled to charge when it is expected to be at home and plugged in:

$$(1 - \hat{\alpha}_{k,t}^{EV}) \cdot P_{k,t}^{EV} = 0 \quad (6)$$

where  $\hat{\alpha}_{k,t}^{EV} \in \mathbb{B}$  is the parameter denoting the presumed availability of the EV.

The main user satisfaction constraint is charging EVs to provide an adequate driving range. Starting from the EV's expected battery SOC on arrival  $\hat{B}_k^{arr} \in [0, B_{max}]$ , the charged energy over all slots  $t \in T$  of the optimisation horizon must accumulate to a full battery SOC,  $B_{max}$ :

$$\hat{B}_k^{arr} + \sum_{t=1}^T \eta \cdot P_{k,t}^{EV} \cdot \Delta t = B_{max} \quad (7)$$

where  $\eta$  is the charging efficiency.

Network constraints include voltage and power flow limits. The phase voltage  $V_{k,t}^{bus}$  at each household  $k$  must be maintained within statutory limits:

$$V_{min} \leq V_{k,t}^{bus} \leq V_{max} \quad (8)$$

The current  $I_{\ell,t}^{line}$  through any cable  $\ell \in L$  may not exceed its rating  $I_{\ell}^{max}$ :

$$I_{\ell,t}^{line} \leq I_{\ell}^{max} \quad (9)$$

The apparent power  $S_t^{tr}$  flowing through the transformer must not surpass the rating  $S_{max}^{tr}$ :

$$S_t^{tr} \leq S_{max}^{tr} \quad (10)$$

#### A. Linear Power Flow Approximation

The values of  $V_{k,t}^{bus}$ ,  $I_{\ell,t}^{line}$  and  $S_t^{tr}$  can be calculated through the well-known power flow equations. Nonlinear power flow equations ensure the most accurate calculation of 3-phase network conditions, but this rapidly complicates the scheduling problem with fine resolutions and large pools of EVs. The linear power flow approximation based on the network sensitivity matrix is adopted here to reduce computational expense and enable use of fast linear programming (LP). Although the sensitivities may not exactly match the continuously varying loads in the network, the approximation is shown to suffice for EV scheduling and, if anything, tends to overestimate the impact of additional EV loads at peak times [13], [22], [23].

The network voltage and line loading sensitivities to changes in load elsewhere can be determined without prior knowledge about EV charging behaviour. They can be calculated by a series of unbalanced three-phase power flow calculations, starting from static load at each household, e.g. average household demand. Iteratively, each household load is increased by 1 kW, with changes in all network voltages and line loading recorded. This produces sensitivity matrices for voltage  $\omega \in \mathbb{R}^{K \times K}$  and line current  $\lambda \in \mathbb{R}^{L \times 3 \times K}$ . Element  $\omega_{i,j}$  denotes the voltage sensitivity, in V/kW, of household  $i$  to changes in power at household  $j$ . Equivalently, element  $\lambda_{\ell,r,j}$  denotes the current sensitivity, in A/kW, of phase  $r$  of line  $\ell$  to changes in power at household  $j$ .

Using the voltage sensitivity matrix  $\omega$ , the voltage constraint equations can be defined as:

$$V_{min} \leq V_{k,t,init}^{bus} \left( \hat{D}_{k,t} \right) + \sum_{j=1}^K \omega_{j,k} \cdot P_{i,t}^{EV} \leq V_{max} \quad (11)$$

where  $V_{k,t,init}^{bus} \left( \hat{D}_{k,t} \right)$  is the voltage at each household and time due to the predicted residential demand  $\hat{D}_{k,t}$  without EV loads. Similarly, the line loading constraints are formulated applying the line loading sensitivity matrix  $\lambda$  to each phase:

$$I_{\ell,r,t,init}^{line} \left( \hat{D}_{k,t} \right) + \sum_{j=1}^K \lambda_{j,r,\ell} \cdot P_{i,t}^{EV} \leq I_{\ell}^{max} \quad (12)$$

where  $I_{\ell,r,t,init}^{line} \left( \hat{D}_{k,t} \right)$  is the current of the respective line, phase and time slot due to the predicted residential demand  $\hat{D}_{k,t}$  without EV loads. As  $V_{k,t,init}^{bus} \left( \hat{D}_{k,t} \right)$  and  $I_{\ell,r,t,init}^{line} \left( \hat{D}_{k,t} \right)$  are not altered by the decision variables, they

are pre-determined using a nonlinear power flow solver before the optimisation cycle.

Employing the linear power flow approximation using network sensitivities, the EV scheduling problem can be quickly solved as an LP. Here, Gurobi is used as a solver and embedded in the simulation framework. Subject to the limitations induced by possible prediction errors and the linear approximation, the solver will produce the optimal coordination of EV charging processes regarding both when and which EVs should charge.

#### IV. APPROACHES TO UNCERTAINTY MITIGATION

As EV charging coordination is embedded in a highly stochastic environment, most decisions may have to be taken before the outturn of uncertain parameters is observed. Optimisation based solely on expected values is not considered robust but can be used to examine sensitivities to forecast deviations in parameters. Consequently, this section introduces uncertainty mitigation options which are then evaluated in Section VI-D. Note that the presented concepts rely on the assumption that probabilities for input parameters can be defined similarly to the modelled distribution functions.

##### A. Availability Uncertainty

The impact of uncertainty in the arrival and departure times of EVs can be reduced by adapting the assumptions regarding when the day-ahead scheduling should assume a vehicle to be available for charging. As outlined in Section II-C, the availability probability is calculated from the CDFs of the arrival and departure time distributions as illustrated in Fig. 5. By default an EV is considered available if the availability probability is equal to or exceeds the expectation value, i.e. 0.5, a 50% chance. However, by defining a threshold  $\nu_\alpha$  for the availability probability above which the day-ahead scheduling should assume a vehicle to be available for charging, the likelihood of an EV being scheduled for charging can be altered. Choosing a higher threshold requires a greater degree of certainty that an EV will be available for it to be scheduled. This raises the robustness to forecast deviations in terms of increasing the likelihood of meeting customer demand for a full battery by charging across a smaller number of periods. The downside is that it narrows the predicted availability period and reduces the flexibility for the charging optimisation as well as potentially raising costs through missing cheap slots where the car actually arrives earlier or departs later than predicted. The adapted availability is given by:

$$\hat{\alpha}_t^{EV} = \begin{cases} 1 & \text{if } \mathbb{P}(\alpha_t^{EV} = 1) \geq \nu_\alpha \\ 0 & \text{else} \end{cases} \quad (13)$$

##### B. Battery Charge Level Uncertainty

Uncertainty mitigation concerning the battery SOC upon arrival in day-ahead scheduling is also based on deviating from the expected value as a forecast value  $\hat{B}^{arr}$ . Using a value that is higher in (say) 70% of cases rather than 50% reduces the risk of not fully charging a battery if the SOC is less than predicted. However, it raises the chance of excessive scheduling if the

SOC is higher than expected for robustness, thereby blocking otherwise spare network capacity that could have been used to charge. The issue is depicted in Fig. 6. Assuming a threshold SOC exceeding  $\nu_B = 0.7$  in all cases instead of its expected value will increase the likelihood of charging being curtailed due to a full battery being achieved prematurely. The effect of prioritising customer satisfaction over reduced capacity and price exploitation is examined in Section VI-D.

#### V. INTEGRATED SIMULATION FRAMEWORK

As outlined previously, in related work, consideration of uncertainties about EV scenarios is more prevalent than recognition of individual uncertainties in mobility patterns. Here, the simulation framework seeks to unite both aspects to properly evaluate the proposed scheduling approach: the optimisation is wrapped in a Monte Carlo (MC) sampling procedure to cope with scenario uncertainty regarding allocation of consumer types and general market conditions.

The simulation routine study is structured as shown in Fig. 7. After retrieving general static parameters such as optimisation horizon, resolution of discrete time steps, and starting time index, different EV scenarios are generated in each iteration through Monte Carlo sampling of the characteristic mobility behaviour distributions introduced in Section II-C. Each scenario refers to the allocation of e.g. certain EV user characteristics in the network, but not to the realisation of uncertainties defined by the user type. As information about voltages and line loading in the distribution network are approximated by linear power flow, corresponding network sensitivities are determined beforehand and are considered valid for all scenarios.

Assessment of a scheduling approach for a specific scenario follows 3 stages: 1. Optimisation: the optimisation schedules and coordinates EV charging according to the selected formulation and uncertainty mitigation options based on the forecasted parameters; 2. Simulation: the simulation uses random deviations from these forecasts in accordance with their uncertainty models and determines outturn performance measures such as charging costs, fulfilment of customer demands, and observation of network constraints relative to the original optimised schedule. Simple heuristic rules are assumed to adjust the scheduling plan during realisation of uncertainty: (i) an EV that is scheduled to charge but not available, will not be charged, and (ii) an EV that has a higher SOC upon arrival than predicted stops charging once the battery is full regardless of any charging scheduled for later. (3) Benchmark: comparing these results to an optimal schedule that could have been achieved with the same algorithm had all parameters been known with perfect information. By comparison with the simulation results, the robustness of the optimisation algorithm to uncertainty can be assessed.

Optimisation algorithms and reference cases are evaluated relative to each other by running the same framework with the same parameters. Because performance analysis of algorithms in a single scenario is inexpensive, the optimisation routine is coupled with a MC simulation as applied in [24], [25]. Arbitrary circumstances may favour one algorithm and bias

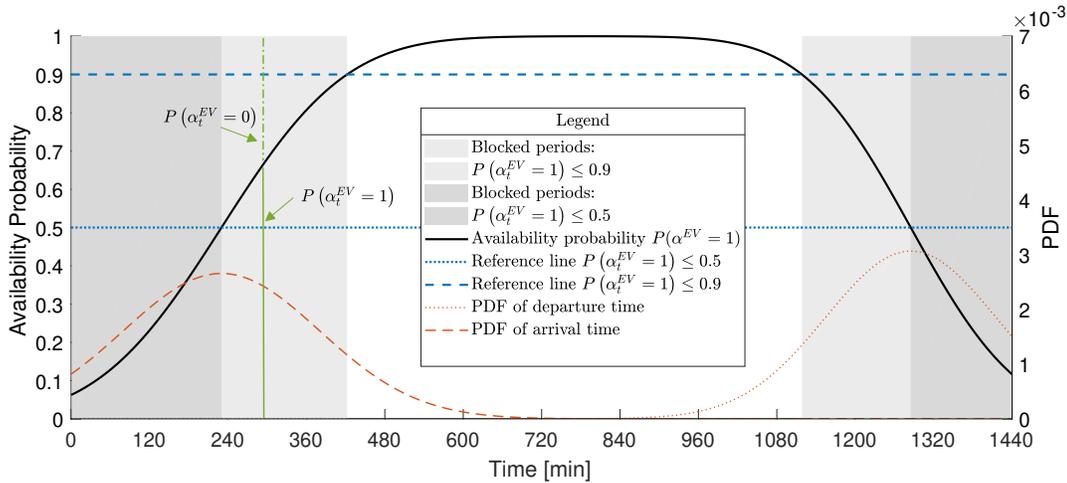


Fig. 5: Approaches for the mitigation of availability uncertainty

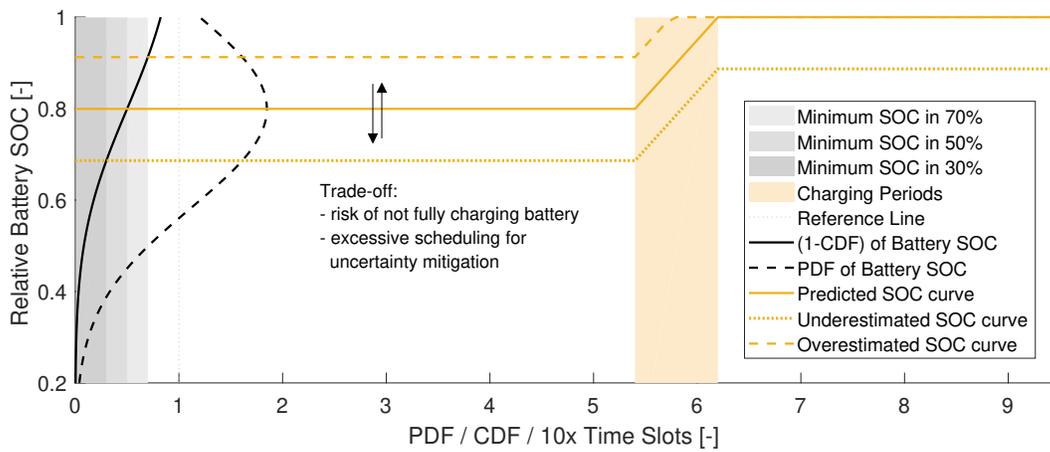


Fig. 6: Mitigation of battery state of charge uncertainty

the evaluation. To guard against this a series of scenarios are generated stochastically forming the basis of comparative performance assessment. Due to a multitude of different user scenarios which the scheduling approach is exposed to, a more accurate evaluation through distributions of performance measures can be attained. Note that here the MC simulation wraps the optimisation cycle and is not a component within the optimisation routine as used e.g. in [26]. The optimisation algorithms are deterministic and stochastic input parameters are entered either with their true expectation values or adapted through more conservative estimates.

The simulation framework has been implemented in Python interfaced with OpenDSS to enable fast time-series three-phase unbalanced power flow studies [27].

## VI. CASE STUDY

### A. Case Description

The IEEE European Low Voltage Test Feeder [28] is used to illustrate the framework, as shown in Fig. 8. This 400 V radial network connects to the MV system through an 0.8 MVA-rated 11/0.4 kV transformer. The network operates at UK voltage standards of +10%/ - 6% at LV and  $\pm 6\%$  at MV.

The test network comprises 55 numbered households connected to single phases. The loads are evenly spread across

the phases. Typical winter weekdays are considered to capture maximum demand in the UK. Individual high-resolution synthetic residential electricity demand profiles were created using the CREST demand model [29]. The aggregate synthetic demand profile compares well with the normalised ELEXON standard load profile for unrestricted domestic customers [30].

It is assumed that an aggregator participates in the wholesale market on behalf of its customers. Regional dynamic tariffs are modelled through reference price data indices from the EPEX SPOT UK exchange market. Prices range over 10 to 40 p/kWh with a significant portion below 22 p/kWh; notably, prices are low late at night and peak during early evening. The aggregator is assumed to be a price-taker.

For simplicity, the model assumes EV specifications from commonly used reference EVs [31] with a consumption rate of  $\zeta = 0.17$  kWh/km, charging efficiency  $\eta = 0.93$ , maximum charging rate of 3.7 kW and a battery capacity  $B_{max} = 30$  kWh for all modelled EVs. Every household has one EV and exclusively charges overnight at home. The charging schedule horizon is 24 hours initiating daily at 1 pm. Charging optimisation is performed at discrete 15 minute time steps assuming constant charging rate within each time slot.

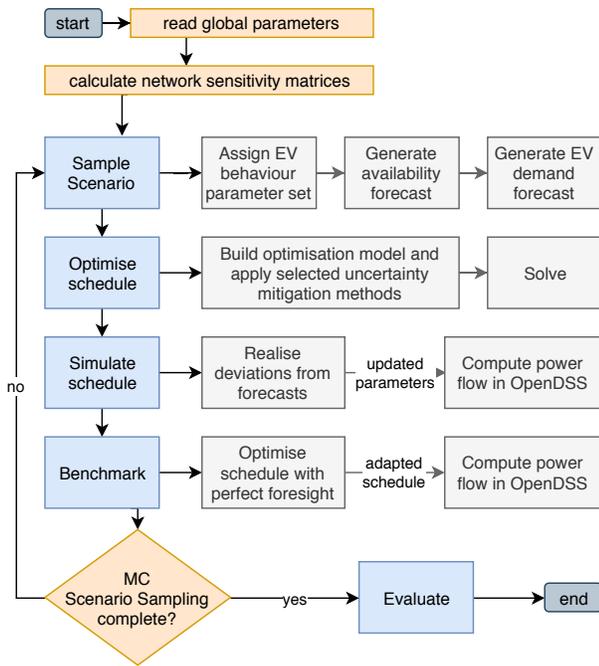


Fig. 7: Flow chart of simulation framework

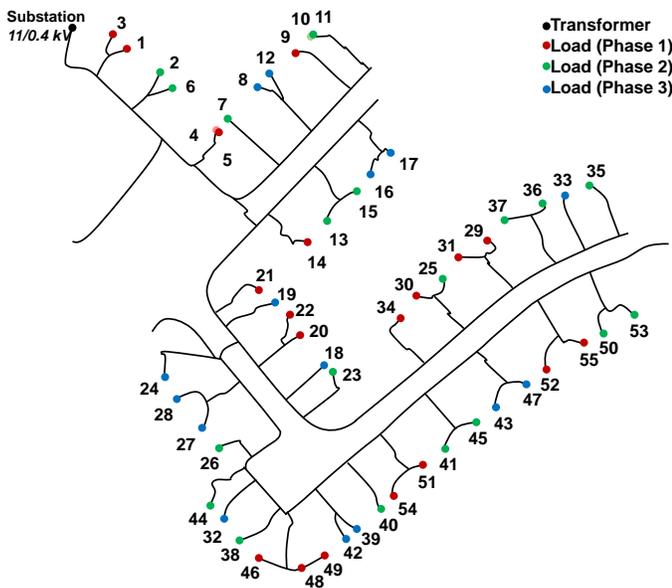


Fig. 8: Topology of IEEE European LV test feeder

### B. Evaluation Criteria and Reference Case

The performance of proposed EV scheduling approaches and mitigation options under uncertainty are evaluated against three factors: (i) economic - relative cost saving of coordinated charging compared to uncontrolled charging; (ii) reliability - satisfaction of demand, i.e. a full battery at the end of the charging period; and (iii) safety - network constraint violations in both severity and frequency.

Several cases are considered. The first is uncontrolled charging as the reference case which refers to the situation where no form of coordinated control is applied, and EVs simply start charging on arrival at the maximum charging rate until their battery is full. This minimises the time to recharge

the battery but disregards economic incentives to defer EV loads. It is a benchmark that is commonly used to evaluate EV scheduling approaches [18], [32]. The second and third cases are optimisation using perfect foresight and under uncertainty, respectively. Following this, the uncertainty mitigation approaches are tested with different threshold parameter values for  $\nu_\alpha$  (chance of vehicle availability) and  $\nu_B$  (battery state of charge level probability) which were introduced in Section IV.

### C. Analysis of Case Study Results

This section illustrates the approach under a single example scenario of household behaviour and a single realisation or outcome for simulation.

Fig. 9 compares uncontrolled charging with optimisation results with perfect information, under uncertainty and with uncertainty mitigation. Uncontrolled charging sees EV charging centred upon periods of high residential demand, thereby causing occasional overloads and voltage violations during simultaneous residential and EV load peaks, particularly around 6:15 pm. The cheaper electricity prices during late night are not exploited. In terms of reliability, uncontrolled charging with its advantage of operating in real-time (and without day ahead uncertainty) can almost guarantee a full battery. In terms of network safety, however, uncontrolled EV charging causes voltage violations and excess loading of around 31%.

Day-ahead coordinated charging schedule with perfect forecast of the realised scenario produces the lowest cost schedule to charge all EVs to full SOC given the availability periods. The realised value of uncertainty parameters, such as EV availability shown in the top-middle in Fig. 9, are directly used in optimisation. There are no instances of line overloading or voltage violation. Because prices may be the same in multiple time slots, different optimal schedules are possible but would ultimately result in the same charging costs for a given case.

Day-ahead coordinated charging schedule under uncertainty ('Opt. Schedule (no mitigation)' in Fig. 9) performs less well than the perfect forecast case. Expected values are used for the uncertainty parameters of EV availability and SOC upon arrival in optimisation. For adjusting the optimised schedule in response to forecast deviations, a simple adjustment rule on the day is assumed. It skips scheduled slots where contrary to the prediction the EV is not available and terminates once the battery is fully charged. In this case, there are few voltage or overloading issues, and the aggregate battery SOC of 95% is reasonably high and covers most daily trip lengths. However, the minimum battery SOC (across the fleet), which is the more critical performance measure influencing customer acceptance, is only 54%. It is evident that uncertainty degrades the ability to schedule optimally.

Considering risk mitigation, day-ahead charging schedule under uncertainty ('Opt. Schedule (with mitigation)' in Fig. 9) shows clear improvement of demand satisfaction, especially regarding the minimal final battery SOC. Rather than expected values of the uncertainty distributions, more conservative estimates are used: to schedule charging the probability of EV availability needs to be higher than 70% and the predicted SOC must not be lower in 70%. The adoption of this joint mitigation parameter set increases the scheduled energy for

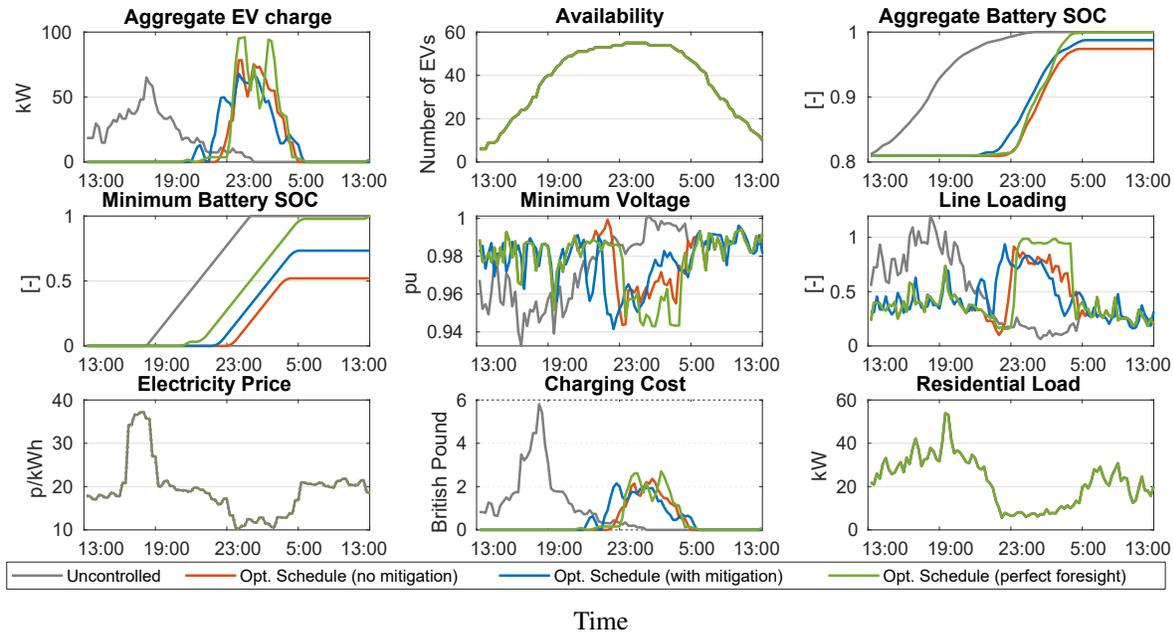


Fig. 9: Comparison of optimised schedules with uncontrolled charging for one example scenario

all EVs. The adjusted scheduling shifts the aggregate charge curve to earlier time slots (top-left in Fig. 9) and some of the cheapest slots are not exploited. The average line loading only slightly increases while voltages deteriorate marginally in this scenario. The next subsection provides an in-depth analysis of the influence of the mitigation options around different conservative estimates of uncertainty parameters and over multiple scenarios of uncertainty realisation.

#### D. Sensitivity Analysis of Uncertainty Mitigation Approaches Across All Scenarios

This section presents the performance of charging algorithms and uncertainty mitigation approaches across multiple scenarios. Uncertainty is represented using 20 scenarios, each representing a typical weekday with different characteristic user behaviour allocated to the households and the variability in performance captured across different situations. To facilitate concise comparison, box-plots are used to indicate distributions (illustrated by quartiles and whiskers) and means (denoted by circles) of the evaluation criteria. Both severity and frequency of network issues are shown. Severity of network constraint violation is quantified by the maximum line loading (relative to line capacity) and the minimum bus voltages (in per unit). Frequency of constraint violations is measured as a ratio of the number of overloading conditions or voltage violations to the total number of corresponding constraints. To aid illustration, the same category of evaluation is represented in the same colours across the figures, e.g. comparing EV demand satisfaction across Fig. 11 and Fig. 12. For evaluations within a figure, different optimisation cases can be distinguished by the shades of colour following the same order in each sub-figure.

The network evaluation among all scenarios for cases with and without EVs under uncontrolled scheduling is given in

Fig. 10 and serves as reference for the following sensitivity analysis. As shown in Fig. 10, without EVs the test network has no active constraints. However, the presence of EV loads causes both voltage and overloading issues in all scenarios. The maximum line overloading across the scenarios averages 30.8% above rating with the minimum voltage 0.02 p.u. below its lower limit of 0.94 p.u. As for violation frequency, about 11.5% of lines experience overloading and 1.5% of total buses have voltage drop beyond limits.

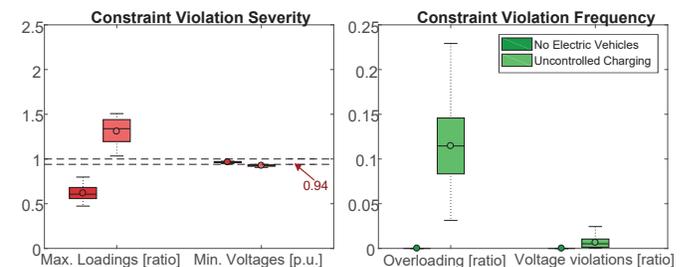


Fig. 10: Network impact of uncontrolled charging across all scenarios

The threshold parameters  $\nu_\alpha$  and  $\nu_B$  vary the degree of conservatism and determine the extent of security margins applied while optimising. The base case for comparison ('LP' in the following figures) was chosen to be the optimisation under uncertainty with expected values.

1) *Vehicle Availability Uncertainty Mitigation*: Fig. 11 shows the impact of changing the availability probability threshold  $\nu_\alpha$  from the original value of 50% to 60%, 70% and 80% on four measures across all scenarios. There is virtually no impact on charging costs or on average battery SOC but minimum battery SOC rises slightly from 54% to 54.8%. A reason for the limited effect is that low electricity prices, and therefore optimised charging, occurs at times of highest

vehicle availability. In terms of network impact, maximum line loading decreases at moderate levels of  $\nu_\alpha$  but then increases when the value reaches 0.8, while voltages progressively fall. This effect is due to limiting charging into fewer time slots at a high level of conservatism.

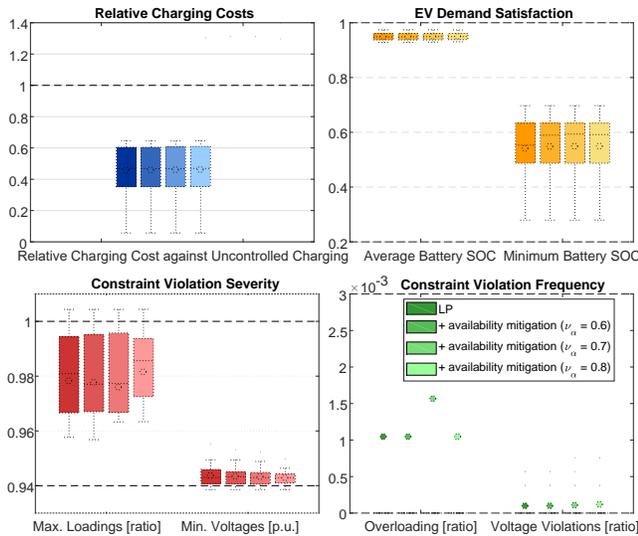


Fig. 11: Sensitivity to  $\nu_\alpha$  across all scenarios

2) *Battery SOC Uncertainty Mitigation*: Fig. 12 shows the effect of raising the threshold  $\nu_B$  of the battery SOC from 50% through to 80% across all scenarios. Increasing security margins increases the reliability of EV demand satisfaction, with the minimum SOC level increasing rapidly. A threshold battery SOC that is not lower in 80% for all cases leads to average SOC levels of more than 98% on average compared to 95% for the threshold of 0.5; more importantly minimum charge levels are raised to 74% rather than 54%. This comes at a cost, with charging costs increasing by 10%. As EVs charging is stopped once full charge is reached according to the assumed real-time adjustment rule, some schedule slots will remain unused and the schedule may not fully coincide with the most economic periods.

A more intelligent control strategy might alleviate this unnecessary cost of day-ahead over-procurement by omitting the most expensive redundant charging slots from the schedule once arrival time and battery SOC is known. Nonetheless, with either type of control, charging will be more expensive as there is a trade-off between granting an EV flexibility and blocking cheap time slots for others. Therefore, the increased satisfaction levels of EVs participating in controlled charging is offset by a suboptimal allocation of finally realised charging patterns.

The flexibility achieved through this form of uncertainty mitigation risks violation of network constraints as EV charging must at least partially be allocated in slots where inhabitants are active and residential electricity demand has more substantial uncertainties.

## VII. DISCUSSION

The paper describes a robust cost-minimising day-ahead scheduling approach for EV charging in LV network in a

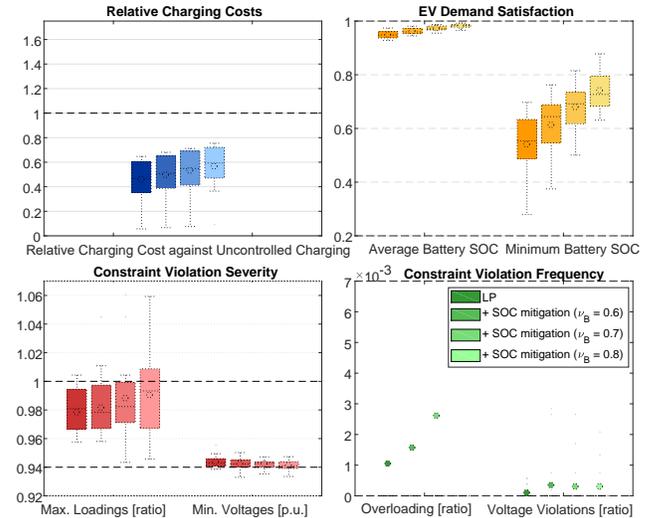


Fig. 12: Sensitivity to  $\nu_B$  across all scenarios

stochastic environment. Models of EV uncertainties complement the development of an optimal scheduling approach with mitigation options for different levels of robustness. The implementation of the proposed coordination algorithm requires communication and control infrastructure. When an EV owner contracts with an aggregator, hardware and/or software would be installed at home to enable remote control and there are real-life trials and deployments with this hardware capability [33], [34].

In the problem formulation, minimum charging cost is selected as the objective function, with available grid capacities being exploited up to their critical limits. This setting aligns with the economic interest of customers and the EV aggregator. This choice sees economic performance as the primary incentive for customers to hand over charging control to an EV aggregator. In principle, the optimisation framework is flexible and can be tailored to use grid reliability (such as minimum voltage and thermal violation) as the objective function within charging cost limits, when the grid reliability outweighs the cost. The optimisation problem minimises overall cost but does not guarantee the lowest cost of each individual vehicle. Individual cost could stand in conflict with overall benefit. However, a redistribution scheme could be used to even out the gain of individual EV users; there is a growing awareness of ‘fairness’ in smart energy systems [35] and a wide body of work on network charging and specifically on fairness in EV charging, e.g. [36], [37] that could be applied.

In day-ahead scheduling, more conservative assumptions reduce the risk of insufficient charge at greater operational cost but ultimately achieving that reduced risk depends on decision making in real time. Improving on the conservative schedules in real-time could be achieved by initialising or increasing the charging rate at time slots where an EV is actually available but was expected not to be. However, increasing unscheduled charging rates may cause network violations in the form of line overloading and voltage issues; mitigating this risk requires real-time power flow calculations or at least an online approximation to manage constraints. While it is feasible to solve such complex real-time network management issues, we

do not seek to address it directly in this study although the split day-ahead/realisation approach seeks to approximate it.

The scheduling of EVs (and other flexible resources in the LV grid) would benefit from accounting for additional sources of uncertainty, such as market price, renewable generation output and no-EV demand forecast. While the scope is restricted in the case study to EV availability and battery SOC for the sake of clarity, the approach can be readily extended to embed more uncertainty factors. The key is to build an appropriate statistical model of such additional uncertainty factors, for instance based on historical data. Modelling these additional uncertainties benefits from them being well-established areas.

There are several promising aspects to extend this work. The paper focuses on day-ahead scheduling with minimal real-time adaptation. Rolling horizon optimisation can be integrated to increase cost savings and robustness towards uncertainties due to more precise and updated predictions. The uncertainty around electricity prices and residential demand is also important to investigate although statistical modelling of the uncertainties and mitigation is more challenging.

### VIII. CONCLUSION

A deterministic robust cost-minimising optimisation for day-ahead EV charging scheduling in residential LV networks has been presented for a stochastic environment. From extensive survey data, a mobility model based on a few continuous distributions was built. Knowledge about the probability distributions of these parameters is used to hedge risks regarding the costs of charging, network overloading, voltage violations, and ensuring desired levels of battery charge levels.

Significant economic and technical improvements are demonstrated compared to uncontrolled charging. Tackling the negative effects of uncertainty through adopting more conservative day-ahead assumptions was considered. More conservative assumptions about EV availability had a limited effect since cost-optimised charging predominantly occurs in periods of comparably certain EV availability. The trade-off between cost and reliability became most evident when addressing battery charge level uncertainty. With increasing security margins, the reliability of EV demand satisfaction rises, particularly the minimum battery charge, while costs increase moderately since vehicles are scheduled to provide more energy than they are expected to require. Overall, valuable insight into uncertainty mitigation of mobility behaviour in EV scheduling was provided.

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